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Assignment #3

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# Problem Solutions Question 1

If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by the formula y = 10t - 1.86t^2.

## Average Velocity Calculations

Interval [1, 2]: **y(1) = 10(1) - 1.86(1)^2 = 10 - 1.86 = 8.14 meters and y(2) = 10(2) - 1.86(2)^2 = 20 - 7.44 = 12.56 meters.** Average Velocity = (12.56 - 8.14) / (2 - 1) = 4.42 m/s

Interval [1, 1.5]: **y(1) = 10(1) - 1.86(1)^2 = 8.14 meters and y(1.5) = 10(1.5) - 1.86(1.5)^2 = 15 - 4.185 = 10.815 meters.** Average Velocity = (10.815 - 8.14) / (1.5 - 1) = 5.35 m/s

Interval [1, 1.1]: **y(1) = 10(1) - 1.86(1)^2 = 8.14 meters and y(1.1) = 10(1.1) - 1.86(1.1)^2 = 11 - 2.26116 = 8.73884 meters.** Average Velocity = (8.73884 - 8.14) / (1.1 - 1) = 5.99 m/s

Interval [1, 1.01]: **y(1) = 10(1) - 1.86(1)^2 = 8.14 meters and y(1.01) = 10(1.01) - 1.86(1.01)^2 = 10.1 - 1.887201 = 8.212799 meters.** Average Velocity = (8.212799 - 8.14) / (1.01 - 1) = 7.28 m/s

Interval [1, 1.001]: **y(1) = 10(1) - 1.86(1)^2 = 8.14 meters and y(1.001) = 10(1.001) - 1.86(1.001)^2 = 10.01 - 1.860186 = 8.149814 meters.** Average Velocity = (8.149814 - 8.14) / (1.001 - 1) = 9.81 m/s

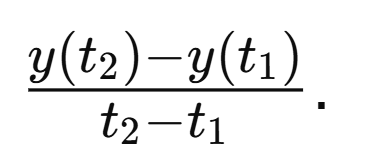
## Instantaneous Velocity Calculation

The formula for the derivative of y with respect to t is y' = 10 - 3.72t. Calculating the derivative at t = 1:

y'(1) = 10 - 3.72 \* 1 = 6.28 m/s.

The Excel file calculates the average velocity of a rock thrown upward on Mars over decreasing time intervals approaching t=1t = 1t=1. By using these intervals, we approximate the instantaneous velocity at t=1t = 1t=1.

**Explanation of Steps:**

1. **Data**: Calculated the height y(t)y(t)y(t) for t=1t = 1t=1 and each t2t\_2t2​ interval (2, 1.5, 1.1, 1.01, 1.001).
2. **Average Velocity**: Computed for each interval as 
3. **Chart**: A line chart visualizes the decreasing average velocities as t2t\_2t2​ approaches t=1t = 1t=1, estimating the instantaneous velocity.

**Question 2**

# Estimation of Instantaneous Velocity of a Particle at t = 1

This document contains the solution to estimating the instantaneous velocity of a particle moving along a straight line, based on the equation of motion given by s(t) = 2sin(πt) + 3cos(πt), where t is in seconds and s is in centimeters.

## Solution Steps

### Part (a): Calculating Average Velocity

The average velocity over a time interval [t1, t2] is calculated as follows:  
  
Average Velocity = (s(t2) - s(t1)) / (t2 - t1)  
  
Using t1 = 1, we calculate this for the following intervals:  
1. [1, 2]  
2. [1, 1.1]  
3. [1, 1.01]  
4. [1, 1.001]  
  
For each interval, we plug in the values of t1 and t2 into the displacement equation, s = 2sin(πt) + 3cos(πt), to calculate s(t1) and s(t2), and then use the average velocity formula.

### Part (b): Estimating Instantaneous Velocity

To estimate the instantaneous velocity at t = 1, we calculate the average velocity over intervals approaching t = 1. This involves computing average velocities for increasingly smaller intervals, which provides an approximation of the instantaneous velocity at t = 1.

## Table: Average Velocity Calculation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| t1 | t2 | s(t1) | s(t2) | Average Velocity |
| 1.0 | 2.0 | -3.0 | 3.0 | 6.0 |
| 1.0 | 1.1 | -3.0 | -3.4712 | -4.712 |
| 1.0 | 1.01 | -3.0 | -3.0613 | -6.1341 |
| 1.0 | 1.001 | -3.0 | -3.0063 | -6.2684 |

**Question 3**

# Estimation of the Limit of f(x) = sin(x) / sin(πx) as x Approaches 0

lim (x -> 0) sin(x) / sin(πx)  
  
We estimate this limit by graphing the function f(x) = sin(x) / sin(πx) and observing its behavior as x approaches 0. The calculations are done in Excel, and the results are provided in this document.

## Solution Steps

### Part (a): Graphing f(x) = sin(x) / sin(πx)

To estimate the limit, we calculated f(x) = sin(x) / sin(πx) for x-values close to 0. The x-values range from -0.1 to 0.1 with an increment of 0.01, allowing us to observe the behavior of f(x) as x approaches 0 from both sides. The calculations and graph are provided in the attached Excel file.

### Part (b): Verifying the Limit by Approaching 0

In Excel, we verify the estimated limit by calculating f(x) for smaller values of x approaching 0 (e.g., 0.01, 0.001, 0.0001). This step helps to confirm the observed limit by examining values closer to 0.

## Table: Calculated Values of f(x) for x Near 0

|  |  |
| --- | --- |
| x | f(x) |
| -0.1 | 0.3231 |
| -0.09 | 0.3222 |
| -0.08 | 0.3213 |
| -0.07 | 0.3206 |
| -0.06 | 0.32 |
| -0.05 | 0.3195 |
| -0.04 | 0.3191 |
| -0.03 | 0.3187 |
| -0.02 | 0.3185 |
| -0.01 | 0.3184 |
| -0.0 | 0.3183 |
| 0.01 | 0.3184 |
| 0.02 | 0.3185 |
| 0.03 | 0.3187 |
| 0.04 | 0.3191 |
| 0.05 | 0.3195 |
| 0.06 | 0.32 |
| 0.07 | 0.3206 |
| 0.08 | 0.3213 |
| 0.09 | 0.3222 |
| 0.1 | 0.3231 |

**Question 4**

# Estimation of the Limit of f(x) = (1 + x)^(1/x) as x Approaches 0

This document provides a solution to estimate the limit:  
  
lim (x -> 0) (1 + x)^(1/x)  
  
The function f(x) = (1 + x)^(1/x) is evaluated for values of x approaching 0. The purpose is to observe the behavior of f(x) as x gets closer to 0 from both sides, positive and negative, and to approximate this limit to five decimal places.

## Solution Steps

### Part (a): Estimating the Limit by Calculation

To estimate the limit, we calculate f(x) = (1 + x)^(1/x) for values of x close to 0. We start with x-values approaching 0 from both positive and negative directions, such as -0.1, -0.01, -0.001, and similar values on the positive side like 0.1, 0.01, 0.001, etc.  
  
As x approaches 0, the values of f(x) converge to a familiar constant, which we approximate to five decimal places.

### Part (b): Illustrating the Limit with a Graph

We plot the function f(x) = (1 + x)^(1/x) in Excel using a range of x-values close to 0, from -0.1 to 0.1 with increments of 0.01. This helps us visualize the behavior of the function as x approaches 0. The graph is included in the Excel file, showing the trend and limit value as x approaches 0 from both directions.  
  
This graphical approach complements our calculations and helps confirm the estimated limit value.

## Table: Calculated Values of f(x) for x Near 0

|  |  |
| --- | --- |
| x | f(x) |
| -0.1 | 2.86797 |
| -0.09 | 2.85165 |
| -0.08 | 2.83565 |
| -0.07 | 2.81996 |
| -0.06 | 2.80459 |
| -0.05 | 2.78951 |
| -0.04 | 2.77472 |
| -0.03 | 2.76021 |
| -0.02 | 2.74597 |
| -0.01 | 2.732 |
| -0.0 | 1.0 |
| 0.01 | 2.70481 |
| 0.02 | 2.69159 |
| 0.03 | 2.6786 |
| 0.04 | 2.66584 |
| 0.05 | 2.6533 |
| 0.06 | 2.64098 |
| 0.07 | 2.62886 |
| 0.08 | 2.61696 |
| 0.09 | 2.60525 |
| 0.1 | 2.59374 |

**Question 5**

# Graphing the Function f(x) = e^x + ln |x - 4|

This document provides a solution to graph the function:  
  
f(x) = e^x + ln |x - 4|  
  
for the interval 0 ≤ x ≤ 5. The function is undefined at x = 4 due to the logarithmic term, which causes a discontinuity in the graph.

## Solution Steps

### Part (a): Graphing the Function in Excel

To graph the function, we calculate values of f(x) = e^x + ln |x - 4| for x-values ranging from 0 to 5, with an increment of 0.1. Since the function is undefined at x = 4 (where ln |x - 4| approaches negative infinity), we exclude x = 4 from the dataset to avoid errors in Excel.  
  
The calculations and resulting graph are included in the attached Excel file, where the function exhibits a steep behavior near x = 4.

### Part (b): Improving the Graph Representation

To obtain a more accurate representation of f(x), especially near the point of discontinuity, we could take finer increments close to x = 4. Alternatively, we could create separate plots for x < 4 and x > 4 to allow Excel to better represent the behavior on each side of the discontinuity.

## Table: Calculated Values of f(x) for x from 0 to 5 (excluding x = 4)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | x | f(x) | | 0.0 | 2.38629 | | 0.1 | 2.46615 | | 0.2 | 2.5564 | | 0.3 | 2.65819 | | 0.4 | 2.77276 | | 0.5 | 2.90148 | | 0.6 | 3.04589 | | 0.7 | 3.20768 | | 0.8 | 3.38869 | | 0.9 | 3.59101 | | 1.0 | 3.81689 | | 1.1 | 4.06888 | | 1.2 | 4.34974 | | 1.3 | 4.66255 | | 1.4 | 5.01071 | | 1.5 | 5.39798 | | 1.6 | 5.8285 | | 1.7 | 6.30686 | | 1.8 | 6.8381 | | 1.9 | 7.42783 | | 2.0 | 8.0822 | | 2.1 | 8.80802 | | 2.2 | 9.6128 | | 2.3 | 10.50481 | | 2.4 | 11.49318 | | 2.5 | 12.58796 | | 2.6 | 13.80021 | | 2.7 | 15.1421 | | 2.8 | 16.62697 | | 2.9 | 18.26946 | | 3.0 | 20.08554 | | 3.1 | 22.09259 | | 3.2 | 24.30939 | | 3.3 | 26.75596 | | 3.4 | 29.45327 | | 3.5 | 32.4223 | | 3.6 | 35.68194 | | 3.7 | 39.24333 | | 3.8 | 43.09175 | | 3.9 | 47.09986 | | 4.1 | 58.0377 | | 4.2 | 65.07689 | | 4.3 | 72.49582 | | 4.4 | 80.53458 | | 4.5 | 89.32398 | | 4.6 | 98.97349 | | 4.7 | 109.5905 | | 4.8 | 121.28727 | | 4.9 | 134.18442 | | 5.0 | 148.41316 | |

**Question 6**

# Numerical Approximation of Limit for f(x) = (x^3 - 1) / (sqrt(x) - 1)

This document provides a solution to estimate the limit:  
  
lim (x -> 1) (x^3 - 1) / (sqrt(x) - 1)  
  
The function f(x) = (x^3 - 1) / (sqrt(x) - 1) is evaluated for values of x approaching 1 from both sides. The goal is to observe the behavior of f(x) as x gets closer to 1 and estimate this limit numerically.

## Solution Steps

### Part (a): Estimating the Limit by Calculation

To estimate the limit, we calculate f(x) = (x^3 - 1) / (sqrt(x) - 1) for values of x close to 1. We use values like 0.9, 0.99, 0.999, 1.001, 1.01, and 1.1 to approximate the behavior of f(x) as x approaches 1. The results and graph are included to visualize the convergence of f(x) towards a limit.

### Part (b): Ensuring Proximity to the Limit

To determine how close x has to be to 1 for f(x) to remain within a distance of 0.5 of the limit, we calculate acceptable values of x near 1 until f(x) is within this range. This process helps identify the interval around x = 1 within which the function values remain close to the estimated limit.

## Table: Calculated Values of f(x) for x Near 1

|  |  |
| --- | --- |
| x | f(x) |
| 0.9 | 5.28093 |
| 0.99 | 5.92531 |
| 0.999 | 5.9925 |
| 1.001 | 6.0075 |
| 1.01 | 6.07531 |
| 1.1 | 6.78156 |